AD A119616

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A119616	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) THE MANN-GRUBBS METHOD		5. TYPE OF REPORT & PERIOD COVERED Technical Report 12/1/81 through 11/30/83
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Nancy R. Mann Frank E. Grubbs		8. CONTRACT OR GRANT NUMBER(s) N00014-82-K-0023
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of California, Los Angeles Dept. of Biomathematics, School of Medicine Los Angeles, CA 90024		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR047-204
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research, Code 411 Arlington, VA		12. REPORT DATE July, 1982
		13. NUMBER OF PAGES 6
14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

DTIC
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SEP 27 1982
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18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

confidence bounds for system reliability; reliability confidence bounds for exponential series systems

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This article was written for the Encyclopedia of Statistical Sciences, edited by Johnson and Kotz. It describes the Mann-Grubbs method of obtaining confidence bounds on system reliability for the exponential series system, for which it was originally designed, and for other models.

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Nancy Mann, Frank Grubbs

Mann-Grubbs Method

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THE MANN GRUBBS-METHOD

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Consider a series system made up of k independent subsystems, each having exponentially distributed failure time T with either no censoring of the subsystem data or only type-II censoring. Testing is without replacement. For this model, system reliability $R_s(t_m)$ at time $t_m > 0$ is equal to

$$\prod_{i=1}^{k} \exp(-t_m \lambda_j) = \exp(-t_m \sum_{j=1}^{k} \lambda_j) , \qquad (1)$$

with $\lambda_j > 0$, $j=1,\ldots,k$, the hazard rate for the jth sub-system. One can determine an upper confidence bound on the system hazard rate $\phi = \sum_{j=1}^{k} \lambda_j$, and from this, of course, can then obtain a lower confidence bound on $R_s(t_m) = \exp(-\phi t_m)$.

The Mann-Grubbs (1972) method for the determination of confidence bounds for system reliability has been adapted to several situations, but was derived originally for this particular model. For this exponential series-system model with type-II censoring, the method yields a lower confidence bound that very closely approximates the lower confidence bound that is most accurate (has the highest probability of being close to the true system reliability) for all values of system reliability, among exact confidence bounds that are unbiased. The restriction of unbiasedness is necessary here because of the nuisance parameters $\lambda_1, \cdots \lambda_k$, the hazard rates for the k

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independent subsystems. For binomial models and exponential

models with censoring by time, the Mann-Grubbs (1974) method

uses an approach similar to that used for the exponential

model with type-II censoring. However, because of discrete-

ness of the data in the binomial case, optimal bounds are

more difficult to approximate. For all of these situations,

however, the method is quite simple to implement.

For the model given by equation (1) with $k = 2$, Lentner

and Buehler (1963) derived the uniformly most accurate un-

biased lower confidence bound on $R_s(t_m)$. Generalization to

$k \geq 2$ was made by El Mawaziny (1965) in his doctoral thesis.

As noted above, their results depend upon the assumption that

for the jth subsystem n_j prototypes have been tested until

r_j, with $1 \leq r_j \leq n_j$, failures occur, $j = 1,\ldots,k$. For the

jth subsystem, one observes the ith smallest failure times

$t_{i,j}$, $i=1,\ldots,r_j$. One then computes the total time on test

for the jth subsystem, $w_j = \sum_{i=1}^{r_j} t_{i,j} + (n_j-r_j)t_{r_j,j}$, $j=1,\ldots,$

k. Calculation of a lower confidence on $R_s(t_m)$ based on the

w_j's and the r_j's by El Mawaziny's method must be performed

iteratively by means of a computer. If both the number of

subsystems and the total number of failures are large, prob-

lems of loss of precision will result. See Mann (1970).

The Mann-Grubbs (M-G) method eliminates the need for a

computer and the problem of loss of precision resulting from

the lengthy calculations. The approach is based on the fact,

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demonstrated by Mann and Grubbs (1972), that the posterior distribution of $\phi = \sum_{j=1}^{k} \lambda_j$ is that of a sum of weighted non-central chi squares. This sum can be well approximated for present purposes by a single weighted chi-square variate with mean m and variance v. Thus, one assumes, using a two-moment fit to chi-square, that $2m\phi/v$ is a chi-square variate with $2m^2/v$ degrees of freedom (see Patnaik, 1949).

The expressions for the conditional mean m and variance v of the system hazard rate derived by Mann and Grubbs for this model have been simplified by Mann (1974) and are given by

$$m = \sum_{j=1}^{k} \frac{(r_j-1)}{w_j} + w_{(1)}^{-1} \qquad (2)$$

and

$$v = \sum_{j=1}^{k} \frac{(r_j-1)}{w_j^2} + w_{(1)}^{-2} \quad , \qquad (3)$$

where $w_{(1)}$ is the smallest of the w_j's.

Once the hazard-function moments have been calculated, then the Wilson-Hilferty transformation of chi-square to normality can be used to facilitate the calculations, since the number of degrees of freedom $\nu = 2m^2/v$ for the approximate chi-square variate $2m\phi/v$ is not generally an integer. To approximate the uniformly most accurate lower confidence bound $\underline{R}_s(t_m)$ on series-system reliability $R(t_m)$ at time t_m and at confidence level $1 - \alpha$ (incorporating the Wilson-Hilferty transformation), one calculates

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$$\underline{R}_s(t_m) = \exp\left[-t_m\, m\left(1 - \frac{v}{9m^2} + \frac{z_{1-\alpha}v^{\frac{1}{2}}}{3m}\right)^3\right] \qquad (4)$$

where z_γ is the 100γth percentile of a standard normal distribution. The Wilson-Hilferty transformation yields an approximation to chi square in this context that, for three or more degrees of freedom, is accurate to within a unit or two in the second significant figure.

For an example of calculation of an approximate confidence bound $\underline{R}_s(t_m)$ on series system reliability, we consider an independent series system containing three subsystems, each with exponential failure time. For each of the three subsystems, prototypes have been life tested with type-II censoring resulting in total-times-on test $w_1 = 42.753$, $w_2 = 45.791$ and $w_3 = 31.890$, with $r_1 = 4$, $r_2 = 3$, and $r_3 = 2$. To obtain $\underline{R}_s(t_m)$ using (2), (3) and (4), one forms

$$m = \frac{4-1}{42.753} + \frac{3-1}{45.791} + \frac{2-1}{31.890} + \frac{1}{31.890} = .17656$$

and

$$v = \frac{4-1}{(42.753)^2} + \frac{3-1}{(45.791)^2} + \frac{2-1}{(31.890)^2} + \frac{1}{(31.890)^2}$$
$$= .00456.$$

Thus, an approximate ninety percent lower confidence bound at $t_m = 1$ is calculated as

$$\underline{R}_s(1) = \exp\left\{-.17656\left[1 - \frac{.00456/.17656^2}{9} + 1.282\sqrt{.00456}/.52968\right]^3\right\}$$
$$= .766 .$$

The El Mawaziny optimal exact lower confidence bound on

$R_s(1)$ is .772. See Mann and Grubbs (1972) or Mann, Schafer,
Singpurwalla (1974).

Schoenstadt (1980) uses simulation to compare an exact
procedure of Lieberman and Ross (1971) and this approximate
method for obtaining lower confidence bounds on system reli-
ability under the model specified by equation (1). He con-
cludes that the results of the simulation runs "seem to
demonstrate the superiority of the M-G bounds in those in-
stances that can be considered of practical importance," and
also demonstrates that the M-G bounds are exact to the
accuracy of his simulation.

For binomial data, the parameter of interest for which a
posterior mean and variance (conditional on the failure data)
are calculated is ξ, the negative of the logarithm of system
reliability R or, for parallel systems, the probability of
system failure, $1 - R$. The parameter ξ has been demonstrated
by Mann (1974a) to have a posterior distribution that is
approximately proportional to a chi-square variate, as does
ϕ in the exponential model. The expressions for the condi-
tional mean and variance of ξ resemble the expressions (2)
and (3) only for models for which *randomized* bounds, commonly
used in binomial models, are obtained.

Expressions for obtaining approximate lower confidence
bounds on R_s or R_p (either randomized or *nonrandomized*) for
the binomial model and for an exponential model with fixed
censoring times can be found in Section 10.4 Mann, Schafer,

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Singpurwalla (1974). Comparisons of results with those of
other methods are also given this for all applicable models,
as well as methods for combining results to obtain bounds for
models more general than simple series or parallel systems.
The latter are also discussed in Mann and Grubbs (1974).

References

El Mawaziny (1967), Ph.D. Thesis, Iowa State University.

Lentner, M.M. and R.J. Buehler (1963), _Journal of the
American Statistical Association_, 670-677.

Lieberman, J. and S. Ross (1971). _Journal of the American
Statistical Association_, 837-840.

Mann, N.R. (1970), _Naval Research Logistics Quarterly_, 41-54.

Mann, N.R. (1974). _Journal of the American Statistical
Association_, 492-495.

Mann, N.R. (1974a), _I.E.E.E. Transactions on Reliability_,
293-304.

Mann, N.R. and F.E. Grubbs (1972), _Biometrika_, 191-204

Mann, N.R. and F.E. Grubbs (1974), _Technometrics_, 335-347.

Mann, N.R., R.E. Schafer and N.D. Singpurwalla (1974),
_Methods for Statistical Analysis of Reliability and Life
Data_, John Wiley and Sons, New York.

Patnaik, P.B. (1949), _Biometrika_, 202-243.

Schoenstadt, A.L. (1980), _Journal of the American Statistical
Association_, 212-216.

Wilson, E.B. and M.M. Hilferty (1931), _Proceedings of
National Academy of Sciences_ (U.S.), 684-688.